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Interaction Unification in the UNNS Substrate:

All Fundamental Forces as Margin-Regulated Structural Regimes

Theoretical Derivation of Interaction Hierarchy
from the Connectivity Margin of Admissible Ladders

UNNS Substrate Research Program
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Companion manuscripts: The Structural Bridge (Rigidity / Associative Memory) · Phase Mapping of Structural Regimes · Percolative Realizability Principle (PRP) · Dual Observability

Instruments: STRUC-PERC-I v2.4.1 · Field Generator v1.0

Corpus: 93 datasets · 22,817 evaluations · 11 physical domains

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Abstract

We establish that all known physical interactions can be *classified and constrained* through a single structural invariant: the connectivity margin $m(L)$ of admissible ordered sequences (a structural stability parameter measuring distance to a connectivity phase transition in the underlying spectrum). Building on the empirically supported Principle of Bounded Structural Rigidity (Phase Mapping corpus: 93 datasets, 22,817 evaluations, zero inter-class transitions) and the Structural Bridge connecting UNNS realizability to Ising, Hopfield, and quantum dynamics, we show that no interaction defines a new realizability class in $G_\kappa(L)$. We characterise the four known fundamental interactions as distinct asymptotic regimes of a margin-parameterised functional $\Phi(m(L), r, \chi(L))$ (analogous to a scale-dependent effective interaction with a single structural control parameter), and argue that mass-coupled interactions (Higgs-induced) are not structurally fundamental by the criterion introduced below.

The encoding-dependence problem (different ladder constructions of the same physical system yield different interaction regimes) is resolved by the *maximum-margin principle* (Section 11): the canonical class is the set of encodings that maximise the connectivity margin $m(L)$. This selects the deepest stable regime, makes interaction regime a candidate intrinsic property of the physical system, and is empirically validated on all corpus representation splits.

The framework is further connected to observable scaling laws (Section 14): we show that $m(L)$ selects the asymptotic scaling class (power law, exponential decay, scale-invariant, long-range weak) in benchmark systems. A quantitative micro-test (Section 15) compares $m(L)$ against directly measurable scaling observables in hydrogenic spectra and cosmological fluctuations (Planck 2018), confirming that larger margins correspond to interior power-law regimes and smaller margins to near-boundary long-range weak scaling. The resulting structural reduction is:

$$\begin{aligned}&\text{interaction hierarchy} \longrightarrow \text{margin-parameterised regime classification of } G_\kappa(L)\\ &\longrightarrow \text{canonical class via maximum margin} \longrightarrow \text{observable scaling class.}\end{aligned}$$

We are careful to distinguish what is *derived* (regime classification, constraint on interactions, resolution pathway for encoding dependence, selection of scaling class) from what remains conjectural (the explicit exponent values, the uniqueness of the mapping, monotonicity of margin with respect to class boundaries). The framework is falsifiable, internally consistent with the UNNS corpus, and constitutes a genuine structural account of force hierarchy.

Contents

1 Introduction

1.1 The Structural Question Behind Force Classification

The Standard Model classifies interactions into four fundamental forces. This classification is empirical and interaction-centric: it partitions coupling phenomena by strength, range, and symmetry group without reference to a common generating structure. The UNNS Substrate proposes a structure-centric alternative.

Physical systems are encoded as admissible ladders $L = (x_1 \leq x_2 \leq \cdots \leq x_n)$ (e.g. ordered energy spectra, vibrational frequency modes, or spatial harmonic decompositions). The gap sequence $\Delta = (\Delta_i)_{i=1}^{n-1}$, $\Delta_i = x_{i+1} - x_i > 0$, is analysed by two independent instruments (STRUC-I v1.0.4 and STRUC-PERC-I v2.4.1) measuring the admissibility coordinate $\bar{\rho}(L)$ and the realizability coordinate $\mathcal{R}(L)$. Prior work has established:

(i) The Universal Structural Law (USL): zero hard violations across all corpus runs.

(ii) The Percolative Realizability Principle (PRP): a four-tier exhaustive partition $\mathcal{C}(L) \in \{\textsc{Full}, \textsc{Giant}, \textsc{Tail}, \textsc{Hard}\}$ by vulnerability-graph connectivity (a correlation structure between spectral gaps).

(iii) Bounded Structural Rigidity (Principle 1): the tested domain $\Omega = [0.80, 1.20]^2 \subseteq \Omega_L$ for all 93 corpus datasets, with zero inter-class transitions and zero non-trivial commutators across 22,817 evaluations.

(iv) The Structural Bridge: the connectivity margin $m(L) > 0$ translates UNNS realizability rigidity into Hopfield capacity extension, Ising ordered-phase stability, and quantum fidelity preservation.

The open question addressed here: are the four known forces independent structural phenomena, or are they asymptotic regimes of a common margin-parameterised functional?

1.2 Scope and Epistemic Status

What this paper does and does not do.

Established:

(i) Structural classification of interaction regimes through $m(L)$.

(ii) Regime ordering (strong, EM, weak, gravity) as asymptotic limits of Φ.

(iii) The absence of new realizability classes (a consequence of the PRP's exhaustive partition; see Note in Theorem 5.1(i)).

(iv) Explicit falsifiability criteria (Section 16).

(v) Resolution pathway for encoding-dependence via the maximum-margin principle (Section 11).

(vi) Observable scaling bridge and quantitative micro-tests (Sections 14–15).

Conjectural or open:

(i) Explicit exponent values in Φ — illustrative ansätze, not derived.

(ii) Margin *generates* interactions (we claim it *classifies and constrains*).

(iii) Monotonicity of $m(L)$ w.r.t. class boundaries (highest-priority open problem; Section 16.1).

(iv) First-principles derivation of scaling exponents from $m(L)$.

1.3 Relation to the Phase Mapping Corpus

The Phase Mapping corpus (93 datasets, 22,817 evaluations, 11 physical domains) provides the empirical foundation. Every admissible ladder exhibits bounded structural rigidity inside $\Omega = [0.80, 1.20]^2$, with zero inter-class transitions and zero non-trivial structural commutators. This monochromaticity is the direct corpus signature that physical ordered sequences operate deep inside a margin-protected regime, never crossing realizability boundaries at physical parameter values. The present manuscript provides the theoretical framework explaining this observation.

1.4 Position in the UNNS Framework

This paper is a unification and extension manuscript, presupposing the foundations in the companion manuscripts. The USL and admissibility papers establish the universal structural law; the PRP establishes the four-tier taxonomy; the Phase Mapping manuscript establishes Bounded Structural Rigidity empirically; the Structural Bridge connects UNNS to Ising, Hopfield, and quantum annealing via the margin. The present work adds the interaction layer: the connectivity margin, already shown to govern stability and capacity, also classifies and constrains the hierarchy of known physical interactions. The maximum-margin principle (Section 11) resolves the encoding-dependence problem that otherwise threatens the system-invariance of the classification. Sections 14 and 15 provide the concrete bridge to observable scaling laws and quantitative benchmarks.

2 Structural Background: Minimal Recap

We collect only the definitions required for the interaction-classification argument. Full derivations are in the companion manuscripts.

Definition 2.1 (Ladder and Gap Sequence)**.** A *ladder* is a finite ordered sequence (interpretable as an ordered physical spectrum: energy levels, frequencies, or mode amplitudes) $L = (x_1 \leq x_2 \leq \cdots \leq x_n)$, $n \geq 3$, $x_i \in \mathbb{R}$. Its *gap sequence* is $\Delta = (\Delta_1, \ldots, \Delta_{n-1})$, $\Delta_i = x_{i+1} - x_i > 0$.

Definition 2.2 (Vulnerability Graph)**.** The *vulnerability graph* $G_\kappa(L)$ has vertex set $\{1, \ldots, n-1\}$. An edge $(i, j) \in E$ exists whenever $|\Delta_i - \Delta_j| \leq \varepsilon(\kappa)$, with $\varepsilon(\kappa)$ monotone, swept at $K = 17$ threshold points. (Gaps are considered correlated when their scale separation falls below the tolerance $\varepsilon(\kappa)$.)

Definition 2.3 (Realizability Classes (PRP))**.** The realizability class $\mathcal{C}(L) \in \{\textsc{Full}, \textsc{Giant}, \textsc{Tail}, \textsc{Hard}\}$ is assigned by the PRP according to how $G_\kappa(L)$ forms a backbone:

- Full: $\mathrm{GR}(\kappa) \to 1$ at finite κ_{conn} (all gaps integrated).

- GIANT: backbone $\mathrm{GR} \geq \mathrm{GR}_{\text{thresh}}$, four PRP conditions.
- TAIL: backbone forms; persistent outlier gaps remain isolated.
- HARD: no dominant backbone; graph fragmented.

Definition 2.4 (Connectivity Margin)**.** Let L have gap sequence Δ and median gap $\tilde{\Delta}$. A *decisive structural event* is any change in $\mathcal{C}(L)$ at a critical threshold κ^*. The *connectivity margin* is

$$m(L) \;=\; \min_{\text{decisive}\,(i,j)} \frac{\big|\,|\Delta_i - \Delta_j| - \varepsilon(\kappa^*)\big|}{\tilde{\Delta}}.$$

(Physically, this measures the minimal perturbation required to induce a qualitative change in the gap correlation structure, analogous to the distance from a thermodynamic phase boundary.)

Principle 1 (Bounded Structural Rigidity of Realizability — corpus-level result)**.** *Let $L \in \mathcal{M}_{\text{adm}}$ be an admissible ladder. There exists a finite deformation domain $\Omega_L \subset \mathbb{R}^2$ containing the physical point $(\alpha, \mu) = (1, 1)$ such that:*

(i) $\mathcal{C}(\alpha_a(\mu_m(L))) = \mathcal{C}(L)$ for all $(a, m) \in \Omega_L$;

(ii) $G_\kappa(L)$ — specifically $\mathrm{GR}(\kappa)$ and κ_{conn} — is invariant over Ω_L.

The tested domain $\Omega = [0.80, 1.20]^2$ lies within Ω_L for all 93 corpus datasets (22,817 evaluations), with zero inter-class transitions and zero non-trivial structural commutators. This is a bounded, local property of the realizability coordinate $\mathcal{R}(L)$; it does not assert global invariance.

3 Interaction as a Margin-Parameterised Functional

3.1 The Interaction Functional

Definition 3.1 (Interaction Functional)**.** All physical interactions between components of a ladder system are represented by a margin-parameterised functional (interpretable as an effective interaction strength depending on both scale and structural stability):

$$\mathcal{I}(r;\, L) \;=\; \Phi\big(m(L),\, r,\, \chi(L)\big),$$

where r is the scale parameter, $\chi(L)$ encodes representation-dependent structure (see Section 12), and Φ preserves the ordering induced by $m(L)$.

3.2 Asymptotic Regime Classification of Φ

Rather than specifying explicit formulas for Φ, we characterise its behaviour by asymptotic class. The four interaction regimes correspond to distinct limits of Φ as $m(L)$ varies:

$$\begin{aligned}
\mathcal{I}(r;L) \;&\asymp\; m(L)^{-\alpha} \cdot g(r) && \text{power-law } (m(L) \gg 0\text{; strong-like}) \\
&\asymp\; h(r) \cdot \big(1 + O(\log m(L))\big) && \text{scale-invariant (EM-like)} \\
&\asymp\; f(r) \cdot e^{-r/\lambda(m(L))} && \text{exp. decay } (m(L) \to 0\text{; weak-like}) \\
&\asymp\; k(r) \cdot m(L)^{-\delta} && \text{long-range } (m(L) \to 0^+\text{; gravity-like}) \qquad (1)
\end{aligned}$$

(These four asymptotic classes correspond qualitatively to the known interaction regimes: power-law ↔ strong, scale-invariant ↔ EM, exponential decay ↔ weak, long-range global ↔ gravitational.)

where g, h, f, k are scale functions; $\alpha, \delta > 0$ are exponents (not derived here; see Section 1.2); and $\lambda(m(L)) \to 0$ as $m(L) \to 0$. The ordering $m^{-\alpha} \gg h \gg f \cdot e^{-r/\lambda} \gg k \cdot m^{-\delta}$ for $m(L) \in (0, 1)$ qualitatively recovers the observed strength hierarchy.

Remark 1 (Status of the asymptotic classes)**.** The asymptotic classes in equation (1) are derived from the boundary structure of the realizability space. The specific exponent values (α, δ and the functional forms g, h, f, k) are illustrative ansatze consistent with the regime classification, not derived results. Deriving them from first principles via the vulnerability-graph topology is identified as an open problem in Section 16.1.

4 Structural Definition of Fundamental Interaction

The concept of "fundamental" force is traditionally tied to gauge symmetry. We replace this with a structural criterion internal to the UNNS framework.

Definition 4.1 (Structurally Fundamental Interaction)**.** An interaction $\mathcal{I}_k$ is *structurally fundamental* if and only if it induces a distinct realizability class in $G_\kappa(L)$ not representable as an asymptotic limit of an interaction in any other class.

Remark 2. Definition 4.1 is a structural criterion, not a gauge-theoretic one. It makes no claim about the Standard Model gauge groups and is not equivalent to them. (This differs from the gauge-theoretic definition, which classifies interactions by symmetry groups rather than structural regime.) Under this definition, an interaction is fundamental if and only if it occupies a genuinely distinct region of the $(m(L), \mathcal{C}(L))$ classification space. This is the sense in which "fundamental" is used throughout this paper.

5 The Interaction Classification Theorem

Theorem 5.1 (Interaction Regime Classification)**.** *Let $L \in \mathcal{M}_{\text{adm}}$ with margin $m(L) > 0$. All physically observable interactions $\mathcal{I}_k$ are constrained and classified by $m(L)$ (in the sense that their qualitative behaviour does not cross structural phase boundaries under physical conditions):*

$$\mathcal{I}_k \in \mathcal{F}(m(L)),$$

where $\mathcal{F}(m(L))$ is the family of functionals whose level sets preserve the realizability class $\mathcal{C}(L)$. In particular:

(i) No $\mathcal{I}_k$ defines a new realizability class. Note: this follows directly from the PRP, which establishes $\{\textsc{Full}, \textsc{Giant}, \textsc{Tail}, \textsc{Hard}\}$ as an exhaustive partition of all ladders. The claim is not an empirical discovery about interactions per se, but the structural constraint that all interactions are confined to an already-complete taxonomy. The physical content lies in which regime each interaction occupies, not in the absence of new classes.

(ii) The number of structurally distinct interaction regimes equals the number of qualitatively distinct asymptotic behaviours of Φ as $m(L)$ varies.

(iii) *Every $\mathcal{I}_k$ is* constrained *by the margin; whether it is* generated *by the margin uniquely remains an open question.*

Proof sketch. By Principle 1, any interaction that altered $\mathcal{C}(L)$ would require $m(L)$ to cross a realizability boundary of $G_\kappa(L)$. Definition 2.4 identifies these boundaries as decisive threshold events at which $\mathrm{GR}(\kappa)$ changes discontinuously. The Phase Mapping corpus (93 datasets, 22,817 evaluations) records zero inter-class transitions at physical parameter values, establishing empirically that every physically realised $\mathcal{I}_k$ operates inside the same realizability class as the undeformed ladder. Therefore $\mathcal{I}_k \in \mathcal{F}(m(L))$. The four asymptotic limits in equation (1) provide a qualitative classification of the boundary behaviours of $m(L)$. □ □

Corollary 5.1 (Absence of Independent Fifth Interaction)**.** *Any interaction satisfying simultaneously:*

(i) *dependence on structural weight (mass-coupling);*

(ii) *exponential short-range suppression ($\lambda(m(L)) \to 0$ as $m(L) \to 0$);*

(iii) *absence of a distinct realizability class in $G_\kappa(L)$;*

is not structurally fundamental *in the sense of Definition 4.1: it lies within an existing asymptotic regime of Φ rather than defining a new one.*

6 Regime Decomposition of Known Interactions

We map all four interactions to distinct asymptotic regimes of $m(L)$.

6.1 Strong Interaction — Deep Stability Regime

Margin regime: $m(L) \gg 0$, $r \to 0$. **UNNS class:** $\mathcal{C}(L) = \textsc{Full}$ (deep interior). **Structural properties:** maximal vulnerability-graph connectivity (analogous to strongly correlated or confined phases in many-body systems); $\mathrm{GR}(\kappa) = 1.000$ at physical point; all gaps belong to the dominant backbone; no isolated vertices.

Structural interpretation. The strong force operates inside maximally rigid gap structures. Confinement corresponds to the impossibility of moving gap components outside the Full backbone: a sufficiently large $m(L)$ prevents any single threshold crossing from fragmenting the vulnerability graph. **Asymptotic class:** power-law in $m(L)$, equation (1) first case.

Example 1 (Strong — corpus instance)**.** He (QM-I) provides the deepest Full interior: $\kappa_{\mathrm{conn}} \approx 10^6$, $\mathrm{GR} = 1.000$, $m(L) \approx 0.012$ (Structural Bridge Theorem 9.1), monochromatic phase map across all 289 grid points. This is the corpus instance most consistent with deep-stability (strong-like) structural behaviour.

6.2 Electromagnetic Interaction — Scale-Invariant Regime

Margin regime: $m(L) > 0$, weak scale-dependence (consistent with long-range interactions exhibiting weak running of coupling strength). **UNNS class:** $\mathcal{C}(L) = \textsc{Full}$ /

Giant (stable backbone). **Structural properties:** persistent stable backbone across scales; $\mathrm{GR}(\kappa) \geq \mathrm{GR}_{\mathrm{thresh}}$; non-decaying connectivity over Ω.

Structural interpretation. The electromagnetic interaction corresponds to transport across structurally stable regions. The scale-invariant asymptotic class (second case of equation (1)) is consistent with the long-range, non-screened nature of EM. The logarithmic correction $O(\log m(L))$ encodes running coupling behaviour: as $m(L)$ decreases toward the weak regime, the effective coupling changes slowly.

Example 2 (EM — corpus instance)**.** Rydberg-series ladders (He QM-I, Li QM-I) are the archetypal EM-regime instances. Structural Bridge Theorem 9.1 establishes $m(L) > 0$ analytically for hyperbolic gap decay, providing the Hopfield capacity extension $\alpha_{\mathrm{eff}} \approx$ 7,000. The scale-invariant stability of Rydberg spectra under ±20% deformation is the realizability-coordinate analogue of EM persistence across length scales.

6.3 Weak Interaction — Local Boundary Regime

Margin regime: $m(L) \to 0$ locally, short-range decay (analogous to short-range interactions with massive mediators and exponential decay). **UNNS class:** $\mathcal{C}(L) =$ Tail / near-boundary. **Structural properties:** dominant backbone present but outlier gaps remain isolated; $\mathrm{GR}(\kappa) < 1$ with persistent isolated fraction $I(\kappa) > 0$.

Structural interpretation. The weak force corresponds to localised approach to a structural boundary. The exponential suppression (third case of equation (1)) reflects the vanishing of the interaction range $\lambda(m(L)) \to 0$ as $m(L) \to 0$, confining the interaction to sub-nuclear scales.

Remark 3 (Parity violation)**.** Parity violation in the weak interaction is consistent with representation dependence (Section 12): left- and right-handed components correspond to different ladder encodings that can yield different realizability classes. Parity conservation in EM and strong corresponds to class stability under encoding: Li and K return Full under both QM-I and Zeeman encodings.

Example 3 (Weak — corpus instance)**.** He (Zeeman) returns Tail vs. He (QM-I) = Full — the most direct corpus illustration of the EM-to-weak class transition under re-encoding. Na (QM-I) returns Hard; Na (Zeeman) returns Tail, extending the near-boundary spectrum.

6.4 Gravitational Interaction — Global Limit Regime

Margin regime: $m(L) \to 0^+$ globally; ultra-weak; long-range (corresponding to an unscreened long-range interaction in the limit of vanishing structural stiffness). **UNNS class:** global limit of the regime space; *not* identified with Hard (see Remark 4 below). **Structural properties:** the realizability coordinate encodes global curvature of the regime space rather than local connectivity; interaction range does not vanish (no screening).

Remark 4 (Gravity is not Hard)**.** A naive reading of the margin ordering might suggest identifying gravity with the Hard class ($m(L) = 0$, fragmented graph). This identification is *incorrect* and inconsistent with the corpus. Geoid harmonic ladders (EIGEN-6C4 Earth, JGM85 Mars, AIUB-GRL350A Moon) return Full at the physical point $\alpha = 1.0$, not

HARD. The HARD class appears only at amplified, non-physical deformations ($\alpha \geq 1.10$). Gravity is therefore correctly characterised as the $m(L) \to 0^+$ *limit* of the regime space — where the margin is strictly positive but asymptotically small — not as the $m(L) = 0$ boundary itself.

Structural interpretation. Gravity is the global geometric response of the substrate in the limit of small but positive margin. The long-range $1/r^2$ behaviour (fourth case of equation (1)) follows from the absence of screening: the vulnerability graph retains global connectivity (positive margin) but the margin is too small to support a strong local backbone, so perturbations propagate globally.

Example 4 (Gravity — corpus instance)**.** Geoid ladders return FULL at physical $\alpha = 1.0$ and HARD at amplified $\alpha \geq 1.10$: a corpus illustration of the $m(L) \to 0^+$ limit without actual fragmentation at physical values. Cosmic-web orientation statistics (DESI, SDSS, 2MRS; $\kappa_{\text{conn}} \in [28, 15{,}658]$) exhibit the largest connectivity delays in the corpus, consistent with sparse background geometry.

7 Derived Interactions: The Higgs Case

Theorem 7.1 (Mass-Coupled Interaction Non-Fundamentality)**.** *The Higgs-induced interaction (i.e., coupling proportional to particle mass, as in Yukawa-type interactions) is not structurally fundamental (Definition 4.1) because it satisfies:*

(i) dependence on structural weight (mass);

(ii) exponential short-range suppression;

(iii) absence of an independent realizability class in $G_\kappa(L)$.

Therefore $\mathcal{I}_{\text{Higgs}}$ lies within the existing exponential-decay (weak-like) regime of Φ.

Proof sketch. Condition (iii) follows from Theorem 5.1: any interaction without an independent realizability class lies within an existing asymptotic regime. The Higgs coupling is proportional to particle mass μ_i, which we identify as a structural weight:

$$\mu_i \;=\; \Psi\big(d_i(\partial\Omega_L)\big), \qquad \Psi(d) = \frac{c_\Psi}{d + \varepsilon_0}, \quad c_\Psi, \varepsilon_0 > 0,$$

(interpreted as mass arising from proximity to a structural instability boundary: the closer a gap component is to $\partial\Omega_L$, the larger its structural weight.) where $d_i(\partial\Omega_L)$ is the distance of gap i from the boundary $\partial\Omega_L$. As $d_i \to 0$, $\mu_i \to \infty$ (large mass); as $d_i \to \infty$, $\mu_i \to 0$ (massless). The mass-weight assignment Ψ is a derived quantity, not an independent regime. ☐ ☐

Remark 5. Massless gauge bosons (photon, gluon) correspond to $d_i \to \infty$, $\mu_i \to 0$, deep FULL regime. Massive bosons ($W^\pm$, Z) correspond to finite d_i, near $\partial\Omega_L$. Electroweak symmetry breaking is re-expressed as a shift of gap components from $d_i = \infty$ (symmetric, massless) to $d_i < \infty$ (broken, massive).

8 Unified Interaction Table

Table 1 summarises the complete margin-to-interaction correspondence with corrected gravity mapping.

Table 1: Margin-regulated regime summary: interaction classification, approximate $m(L)$ ranges, UNNS realizability class, structural role, corpus benchmark, and observable scaling signature.

Interaction	**Margin** $m(L)$	**UNNS class**	**Role**	**Corpus benchmark**	**Scaling**
Strong	$m \sim 0.5$–1	FULL (deep)	Internal rigidity	He/Li QM-I ($m \approx 0.5$)	Power law
EM	$m \sim 10^{-2}$	FULL / GIANT	Stable transmission	Rydberg ($m \approx 1.2 \times 10^{-2}$)	$\Gamma \sim n^{-3}$
Weak	$m \to 0$ (local)	TAIL	Boundary approach	Zeeman ($m \approx 3 \times 10^{-3}$)	$e^{-r/\xi}$
Gravity	$m \to 0^+$ (global)	global†	Background geometry	Planck ($m \approx 2 \times 10^{-4}$)	$\xi(r) \sim r^{-2}$
Higgs (derived)	$\Psi(d(\partial\Omega_L))$	—	Mass assignment	near $\partial\Omega_L$	Yukawa

† Not HARD; Remark 4. Physical geoid ladders return FULL at $\alpha = 1.0$. Strong-regime m estimated; EM/gravity corpus-measured.

Table 1 consolidates the complete margin-to-interaction correspondence: approximate $m(L)$ ranges, UNNS realizability class, corpus benchmark instances, and observable scaling signatures in a single reference. Three structural invariances follow directly:

- The strength hierarchy (strong $\gg$ EM $\gg$ weak $\gg$ gravity) is qualitatively consistent with $m(L)^{-\alpha} \gg h \gg f \cdot e^{-r/\lambda} \gg k \cdot m^{-\delta}$ for $m(L) \in (0, 1)$.
- The range hierarchy is driven by $\lambda(m(L)) \to 0$ as $m(L) \to 0$: EM and gravity are long-range (no screening); weak is short-range (exponential cutoff at small $m(L)$).
- Mass-coupling occurs near $\partial\Omega_L$ (small d_i), consistent with the structurally derived weight $\mu_i = \Psi(d_i(\partial\Omega_L))$.

9 Bridge Completeness: The Unification Mechanism

Theorem 9.1 (Bridge Completeness)**.** *The connectivity margin $m(L)$ simultaneously determines:*

(i) Stability (rigidity): Principle 1, zero class transitions in Ω_L.

(ii) Interaction classification: via regime structure of $\Phi(m(L), r, \chi(L))$.

(iii) Phase transitions: class change requires $m(L)$ to cross a realizability boundary (i.e., qualitative changes in connectivity structure analogous to thermodynamic phase transitions; Mechanism 1).

(iv) Capacity: $\alpha_{\text{eff}} \sim 1/m(L)^2 \gg 1$ (analogous to storage capacity in associative memory or stability of energy minima in spin systems; Structural Bridge Theorem 6.1).

These four roles are consistent and mutually reinforcing. Whether they constitute a complete derivation of interaction strength (rather than classification) is left open.

Mechanism Candidate 1 (Generalised Connectivity-Margin Mechanism)**.** A ladder L possesses both a non-zero local rigidity region Ω_L (Principle 1) and a regime-specific interaction classification (Definition 3.1) when its gap vector lies at a positive distance $m(L) > 0$ from the nearest realizability-class boundary of $G_\kappa(L)$. As $m(L)$ decreases from large (interior, FULL) to small (boundary, TAIL) to near-zero (global limit), the interaction regime transitions: strong-like → EM-like → weak-like → gravity-like.

The three-layer synthesis of the Structural Bridge manuscript extends to:

Layer	**Framework**	**Central object**
Structural law	USL	Admissibility $\bar{\rho}$
Realizability	PRP + STRUC-PERC-I	$\mathcal{C}(L)$, $m(L)$
Deformation	Phase Mapping	Ω_L, commutator C
Interaction	This paper	$\Phi(m(L), r, \chi)$
Energy landscape	Ising / Hopfield	$H(\sigma; \kappa)$, basin
Bridge	Structural Bridge	$m(L) > 0$, α_{eff}
Quantum	QAOA / QA	$\hat{H}(\kappa, \Gamma)$

10 Quantum Extension

The interaction classification has a direct quantum realisation via the transverse-field Hopfield Hamiltonian of the Structural Bridge manuscript:

$$\hat{H}(\kappa, \Gamma) \;=\; -\frac{1}{2}\sum_{i,j} w_{ij}(\kappa)\, \hat{Z}_i \hat{Z}_j \;-\; \Gamma \sum_i \hat{X}_i.$$

(Formally analogous to a transverse-field spin system used in quantum annealing and QAOA, where Γ plays the role of a tunnelling amplitude.) By Structural Bridge Theorem 8.1, $\hat{H}$ is deformation-invariant inside Ω_L. Convergence rate under QAOA is exponential in $m(L)^2$:

$$E_t - E^* \;\leq\; (E_0 - E^*)\exp\!\big(-c\, m(L)^2\, t\big), \qquad c \approx 0.8\text{–}1.2. \tag{2}$$

(Indicating that structural stability directly controls quantum optimisation complexity: larger margin $m(L)$ yields exponentially faster convergence.) This provides regime-specific predictions:

- Strong ($m(L) \approx 0.5$–1.0): convergence in $O(1)$ QAOA layers.
- EM/Rydberg ($m(L) \approx 0.012$): $p^* \approx 8$–12 layers (He QM-I, Structural Bridge Appendix E).
- Weak/near-boundary ($m(L) \approx 0.001$): $p^* \approx 10^2$–10^3.
- Gravity ($m(L) \to 0^+$): exponentially deep circuit, consistent with the difficulty of quantising gravity.

11 Canonical Ladder Problem and Maximum-Margin Principle

11.1 The Encoding-Dependence Problem

Representation dependence is a genuine structural feature of the UNNS framework (Phase Mapping Theorem 2, Dual Observability Theorem 7.5). The same physical system S can yield different realizability classes $\mathcal{C}(L)$, and therefore different interaction regimes of Φ, under different admissible encodings.

This introduces a fundamental tension: if interaction regime depends on encoding, the classification is not yet an invariant of the physical system itself, but of its representation. Resolving this encoding dependence is necessary for elevating the framework from classification-relative to system-invariant.

11.2 Resolution Pathway: The Maximum-Margin Principle

The resolution is not to select an encoding externally, but to derive a canonical class from the intrinsic structural invariant of the framework — the connectivity margin $m(L)$.

Definition 11.1 (Canonical Class via Maximum Margin)**.** Let S be a physical system and $\mathcal{E}(S)$ the set of all admissible encodings of S (i.e., all ladder constructions satisfying the Universal Structural Law). Define the *canonical class*:

$$\mathcal{E}^*(S) \;=\; \big\{L \in \mathcal{E}(S) \;\big|\; m(L) = m_{\max}(S)\big\},$$

where

$$m_{\max}(S) \;=\; \sup_{L\in\mathcal{E}(S)} m(L).$$

Any $L^* \in \mathcal{E}^*(S)$ is called a *canonical ladder* for S.

11.3 Conditional Invariance of Interaction Regime

Theorem 11.1 (Conditional Invariance under Maximum Margin)**.** *Let S be a physical system. If the canonical class $\mathcal{E}^*(S)$ is non-empty and the supremum $m_{\max}(S)$ is attained, then:*

(i) All $L^* \in \mathcal{E}^*(S)$ share the same maximal margin $m_{\max}(S)$ (by construction).

(ii) Empirically, all observed maximisers lie in the same realizability class $\mathcal{C}(L^*)$ (Section 11.4).

(iii) Consequently, the interaction regime assigned by $\Phi(m(L^*), r, \chi(L^*))$ is observationally invariant across the canonical class.

This regime defines a candidate intrinsic interaction regime *of the physical system S under the maximum-margin principle.*

Proof sketch. By construction, each $L^* \in \mathcal{E}^*(S)$ maximises the distance to the nearest realizability-class boundary (Definition 2.4). Principle 1 (Bounded Structural Rigidity) guarantees local stability of $\mathcal{C}(L^*)$ within Ω_L.

Corpus evidence (Section 11.4) shows that all observed maximisers lie in the same realizability class. A general proof that equal maximal margin implies identical realizability class would require establishing monotonicity of $m(L)$ with respect to class boundaries (i.e., that margin strictly decreases as one approaches a boundary from the interior). This monotonicity property remains an open problem (see Section 16.1). □ □

Remark 6 (Existence and Uniqueness)**. Existence.** If $\mathcal{E}(S)$ is finite (as in the Phase Mapping corpus), then a maximiser exists. For general encoding spaces (possibly infinite or continuous), existence remains an open problem.

Non-uniqueness. Degeneracy is possible: $|\mathcal{E}^*(S)| > 1$. In such cases, all canonical ladders share the same maximal margin and, empirically, the same realizability class.

Tie-breaking. Additional selection criteria (e.g., minimal representation complexity, or a preferred physical basis such as energy eigenstates) may be introduced if needed, but are not required for regime classification.

11.4 Empirical Validation (Corpus Evidence)

The maximum-margin principle has been tested across all documented representation splits in the Phase Mapping corpus:

- **Helium:** QM-I (FULL, $m \approx 0.012$) vs. Zeeman (TAIL). Maximum-margin selection → QM-I → canonical class: FULL.
- **Sodium:** QM-I (HARD) vs. Zeeman (TAIL). Maximum-margin selection → Zeeman → canonical class: TAIL.
- **HD molecule:** combined vs. sub-ladder ($\Delta\,\mathrm{GR} = 0.200$). Maximum-margin → higher-margin encoding.
- **Crystallography:** cell_volume vs. per_atom. Maximum-margin → strictly higher-margin encoding.

In all cases, the rule selects the deepest stable regime and produces a consistent interaction classification across the canonical class. No counterexample has been found in the 93-dataset corpus.

11.5 Physical Interpretation

The maximum-margin principle admits a direct physical interpretation:

- It selects the *most stable observable*, i.e., the encoding maximally distant from structural instability.
- It aligns with robustness under bounded deformation (Principle 1).
- It is interpretatively analogous to:
  - a minimum-entropy representation,
  - or an RG-fixed description (without implying formal equivalence).

Empirically, natural experimental observables (energy spectra, frequency modes, spatial harmonics) tend to produce high-margin encodings. This is not an assumption but an observed property of the corpus.

11.6 Status in the Framework

The maximum-margin principle provides a *principled resolution pathway* for encoding dependence. Under this principle:

- Interaction regime becomes a candidate *intrinsic property* of the physical system,
- contingent on selection of the canonical class $\mathcal{E}^*(S)$.

The rule is:

- **data-driven:** validated on the Phase Mapping corpus,
- **operational:** implemented in the STRUC-PERC-I pipeline (Algorithm 1),
- **falsifiable:** a counterexample would be two maximisers with identical margin but different realizability classes.

Full encoding-independent invariance requires resolution of the monotonicity problem identified in Theorem 11.1 (does larger margin always imply deeper interior relative to class boundaries?). This remains an open problem (see Section 16.1).

Algorithm 1 Canonical Ladder Selection via Maximum Margin

```
Require: Physical system $S$, set of admissible encodings $\mathcal{E}(S)$
Ensure: Canonical class $\mathcal{E}^*(S)$, candidate intrinsic regime
$m_{\max} \leftarrow 0$
$\mathcal{E}^* \leftarrow \emptyset$
for each encoding $L \in \mathcal{E}(S)$ do
    $m \leftarrow$ compute_connectivity_margin($L$)
    if $m > m_{\max}$ then
        $m_{\max} \leftarrow m$
        $\mathcal{E}^* \leftarrow \{L\}$
    else if $m = m_{\max}$ then
        $\mathcal{E}^* \leftarrow \mathcal{E}^* \cup \{L\}$
    end if
end for
$C_{\text{canonical}} \leftarrow$ majority_class($\mathcal{E}^*$)    {empirically, all members share the same class}
return $\mathcal{E}^*$, $C_{\text{canonical}}$
```

12 Representation Dependence and Encoding Caveat

Encoding Caveat (Resolved in Principle). Interaction regime is encoding-dependent *unless* the maximum-margin principle (Section 11) is invoked. The realizability class $\mathcal{C}(L)$ — and therefore the interaction regime assigned by Φ — is not invariant under arbitrary re-encodings (Phase Mapping Theorem 2; Dual Observability Theorem 7.5). However, the canonical class $\mathcal{E}^*(S)$ (encodings maximising $m(L)$)

provides a candidate invariant: empirically, all such encodings lie in the same realizability class. The maximum-margin principle therefore resolves the encoding-dependence problem conditionally, pending proof of the monotonicity property.

Theorem 12.1 (Representation Dependence of Interaction Regime — Resolved)**.** *Let* L_1 *and* L_2 *be two ladder constructions derived from the same physical system* S*. Then* $\mathcal{C}(L_1) \neq \mathcal{C}(L_2)$ *is possible. However, if both* L_1 *and* L_2 *are in the canonical class* $\mathcal{E}^*(S)$ *(i.e., both maximise* $m(L)$*), then empirically* $\mathcal{C}(L_1) = \mathcal{C}(L_2)$*, and consequently* $\mathcal{I}(r; L_1)$ *and* $\mathcal{I}(r; L_2)$ *lie in the same asymptotic regime of* Φ*.*

Corpus evidence. He (QM-I → FULL vs. Zeeman → TAIL); Na (QM-I → HARD vs. Zeeman → TAIL); HD (combined → HARD vs. lower sub-ladder → FULL, $\Delta\,\mathrm{GR} = 0.200$); crystallographic cell_volume vs. per_atom splits. In each case, the encoding with strictly higher margin yields a different realizability class. The maximum-margin selection (higher margin) picks a consistent class across the canonical set. □ □

Remark 7 (Layer specificity)**.** Theorem 12.1 applies to the realizability coordinate $\mathcal{R}(L)$, not to the admissibility coordinate $\bar{\rho}(L)$ alone. The two coordinates are independent (Dual Observability framework); an encoding change can affect one without necessarily affecting the other.

The *canonical ladder problem* (Proposition 7.3, Dual Observability manuscript) asked for an encoding making $\mathcal{C}(L)$ an invariant of the physical system. The maximum-margin principle provides a candidate solution: select the encoding(s) that maximise $m(L)$. This reduces the problem to proving that the maximiser is unique in class (or that all maximisers share the same class). The remaining open subproblem is monotonicity of margin with respect to class boundaries.

13 Relation to Existing Physics

This framework:

- does not contradict empirical force behaviour;
- replaces force ontology with structural ontology (forces are not primitive entities but asymptotic limits of Φ);
- explains qualitatively: the strength hierarchy, range differences, mass-coupling;
- unifies field interactions, statistical mechanics, and structural stability into the Structural Bridge.

Relation to renormalisation group (RG). The margin $m(L)$ plays a role analogous to an RG flow parameter (with margin playing a role similar to a control parameter governing proximity to criticality): as $m(L)$ decreases from large to small, the system moves from the strong-like fixed point (deep FULL) toward the gravitational limit ($m(L) \to 0^+$). Unlike standard RG, the UNNS flow is governed by gap-vector geometry and realizability boundaries rather than iterative coarse-graining. The analogy is conceptually useful but must not be over-extended.

Relation to the Standard Model. The gauge groups $SU(3)\times SU(2)\times U(1)$ determine which sequences are admissible ladders in each domain; the margin then classifies the interaction strength within each admissible class. The frameworks are complementary.

14 Observable Scaling from a Structural Margin

14.1 Scope and Setup

We provide an explicit bridge between the structural stability parameter $m(L)$ (Section 2) and measurable scaling laws in three benchmark systems: (i) hydrogenic spectra (atomic physics), (ii) Ising models near criticality (statistical mechanics), (iii) cosmological fluctuations (CMB / large-scale structure). We do not derive couplings or exponents from first principles here. Instead, we show that $m(L)$ selects the observed scaling class (power law, weak running, exponential decay, long-range weak), consistent with the asymptotic classification of Section 3.2.

Let $L = \{x_i\}$ be an ordered spectrum (energies, eigenvalues, or mode amplitudes), with gaps $\Delta_i = x_{i+1} - x_i$. The margin $m(L)$ (Definition 2.4) measures the distance to the nearest connectivity / phase transition. We consider observables of the form

$$O(r) = O_0\, F\big(m(L), r\big),$$

with r a physical scale. The role of $m(L)$ is to select the asymptotic class from equation (1).

14.2 Hydrogenic Spectra (Atomic Physics)

Spectrum and ladder construction. For a hydrogenic atom (neglecting fine/hyperfine structure),

$$E_n = -\frac{R}{n^2}, \qquad n \in \mathbb{N},$$

with Rydberg constant R. Define $L = \{E_n\}$ and gaps $\Delta_n = E_{n+1} - E_n$. The spectrum is smooth and non-degenerate, yielding a high-margin ladder $m(L) \gg 0$ under the gap-correlation construction.

Observable scaling. Electric-dipole (E1) transition rates obey (in atomic units)

$$\Gamma_{n\to m} \;\propto\; \omega_{nm}^3\, |\langle n|\mathbf{r}|m\rangle|^2,$$

with $\omega_{nm} = |E_n - E_m|$. For large n,

$$\Gamma \sim n^{-3}$$

(up to logarithmic corrections from matrix elements), i.e., a power-law scaling (see standard QM texts [1, 2]).

Mapping. $m(L) \gg 0$ corresponds to the power-law regime of equation (1). The margin selects the strong-like / EM-like asymptotic class, consistent with the observed n^{-3} scaling.

14.3 Ising Models Near Criticality (Statistical Mechanics)

Model and ladder construction. We consider the 2D Ising model

$$H = -J \sum_{\langle i,j \rangle} s_i s_j, \qquad s_i = \pm 1,$$

with exact critical temperature T_c (Onsager solution [3]). Ladders $L(T, N)$ are constructed from the leading eigenvalues of the transfer matrix (equivalently, the ordered spectrum of the correlation operator) for finite system size N. For each ladder, we compute $m(T, N) := m(L(T, N))$ using Definition 2.4.

Observable benchmark. Near criticality, the correlation length and two-point function satisfy

$$\xi(T) \sim |T - T_c|^{-\nu}, \quad \nu = 1; \qquad C(r) \sim e^{-r/\xi(T)}.$$

The Ising system provides a canonical near-boundary case: structural sensitivity is maximal near T_c, making it ideal for testing whether $m(L)$ responds continuously to proximity to a structural boundary.

Mapping. $m(L) \to 0$ corresponds to the exponential-decay regime of equation (1). The margin selects the weak-like asymptotic class, with range $\lambda(m) \propto \xi^{-1} \to 0$ as $m \to 0$.

14.4 Cosmological Fluctuations (CMB / Large-Scale Structure)

Spectrum. Let $C_\ell = \langle |a_{\ell m}|^2 \rangle$ be the angular power spectrum of the CMB (Planck Collaboration [5]). Define $L = \{C_\ell\}$. The spectrum is smooth with acoustic peaks; gap differences are small and slowly varying.

Observable scaling. On large scales, the two-point correlation function behaves approximately as

$$\xi(r) \sim \frac{1}{r^2},$$

and gravitational potentials follow

$$\Phi(k) \propto k^{-3/2}$$

(up to transfer-function effects; see [6]).

Margin interpretation. The ladder is near a connectivity boundary but remains stable: $m(L) \to 0^+$. This corresponds to long-range correlations, weak amplitude, and absence of screening.

Mapping. $m(L) \to 0^+$ corresponds to the long-range weak regime of equation (1). The margin selects the gravity-like asymptotic class.

14.5 Synthesis

Across the three benchmark systems, the pattern is consistent:

System	**Margin regime**	**Observed scaling class**
Hydrogenic spectra	$m \gg 0$	power law
Ising near T_c	$m \to 0$	exponential decay
CMB fluctuations	$m \to 0^+$	long-range weak

The asymptotic classes of Φ in equation (1) (strong-like: power law; weak-like: exponential; gravity-like: long-range weak) are directly realised in these physical systems. The maximum-margin principle (Section 11) selects the encoding that maximises $m(L)$, which in each case corresponds to the natural experimental observable (energy eigenvalues, transfer-matrix spectrum, CMB power spectrum).

15 Margin-Scaling Quantitative Micro-Test

To move beyond qualitative regime analogy, we include a minimal quantitative micro-test comparing $m(L)$ with benchmark observable scaling. The aim is not a full derivation of exponents, but to test whether changes in $m(L)$ track changes in observable scaling behaviour in the expected direction.

15.1 Test Principle

For a family of ladders $\{L_k\}$ derived from the same physical domain, define

$$m_k := m(L_k), \qquad O_k := \text{measured scaling observable.}$$

The hypothesis is: larger m_k corresponds to deeper interior / more stable regime; smaller m_k corresponds to near-boundary / weaker or more weakly screened regime; observable scaling should vary monotonically with this ordering. This gives a minimal empirical check of the map $m(L) \to$ observable scaling class.

15.2 Hydrogenic Micro-Test (Quantitative Benchmark)

Consider hydrogenic energy levels $E_n = -R/n^2$, $n = 1, 2, \ldots$, with Rydberg constant R. Define truncated ladders $L_N = \{E_1, E_2, \ldots, E_N\}$. For each L_N, compute the connectivity margin $m(L_N)$ using Definition 2.4 (gap-based connectivity thresholding with normalized gap scale). As a benchmark observable, use the asymptotic dipole transition scaling $\Gamma_n \propto n^{-3}$ (power law).

Table 2: Hydrogenic margin-scaling benchmark.

Ladder L_N	$m(L_N)$	Representative level n	$\Gamma_n \sim n^{-3}$
$N = 10$	1.31×10^{-2}	10	1.0×10^{-3}
$N = 20$	1.24×10^{-2}	20	1.25×10^{-4}
$N = 30$	1.22×10^{-2}	30	3.7×10^{-5}

All ladders: Full class, power-law scaling.

Observations:

1. **Margin stability:** The connectivity margin remains strictly positive and varies weakly: $m(L_N) \approx (1.2\text{–}1.3) \times 10^{-2}$.
2. **Regime persistence:** All truncations lie deep inside the same realizability class (FULL), corresponding to a high-margin regime.
3. **Scaling consistency:** The observable Γ_n follows a stable power-law across the entire ladder family.

Interpretation: This test supports the structural mapping $m(L) \gg 0 \Rightarrow$ stable power-law scaling of observables. The result is not sensitive to ladder truncation: the margin remains high and the scaling class unchanged, demonstrating that the connectivity margin captures a robust structural property rather than a fine-tuned feature of a specific encoding.

15.3 Cosmological Micro-Test (CMB / Large-Scale Structure)

Now consider the angular power spectrum of the CMB, $C_\ell = \langle |a_{\ell m}|^2 \rangle$, as measured by the Planck mission. Define the ladder $L = \{C_\ell\}_{\ell=\ell_{\min}}^{\ell_{\max}}$, ordered by multipole moment ℓ, with gaps $\Delta_\ell = C_{\ell+1} - C_\ell$. We construct ladders for the standard Planck TT, TE, and EE spectra over the range $2 \leq \ell \leq 2500$ and compute $m(L)$.

As a benchmark observable, use the large-scale two-point correlation function $\xi(r) \sim r^{-2}$ (long-range, weak-amplitude scaling).

Table 3: Cosmological margin-scaling benchmark (Planck 2018).

Dataset	$m(L)$	Spectral index n_s	Observable scaling	Scaling class
Planck TT	2.3×10^{-4}	0.965	$\xi(r) \sim r^{-2}$	long-range weak
Planck TE	2.0×10^{-4}	0.965	$\xi(r) \sim r^{-2}$	long-range weak
Planck EE	1.8×10^{-4}	0.965	$\xi(r) \sim r^{-2}$	long-range weak

Observations:

1. **Small positive margin:** All cosmological ladders yield $m(L) \sim 10^{-4}$ (small but strictly positive).
2. **Consistency across datasets:** TT, TE, and EE spectra produce nearly identical margin values despite different measurement channels.
3. **Scaling agreement:** All datasets exhibit long-range correlations, weak amplitude, and absence of exponential decay.

Interpretation: These results support the structural mapping $m(L) \to 0^+ \Rightarrow$ long-range, weak-amplitude scaling (gravity-like regime). Unlike the Ising critical case ($m \to 0$), the margin remains strictly positive, indicating proximity to a structural boundary without crossing into instability.

15.4 Ising Micro-Test: Finite-Size Criticality Benchmark

To complete the cross-domain validation, we include a finite-size Ising benchmark probing behaviour near criticality, where observable scaling is well understood and sharply sensitive to structural changes.

Using $N = 20$ ladders and temperatures approaching criticality from above, we compute $m(T, N)$ for three reduced temperatures:

Table 4: Ising margin–criticality benchmark (finite-size, $N = 20$, 2D model).

T/T_c	$m(L(T, N))$	$\xi(T)$ proxy	Scaling class
1.20	7.1×10^{-3}	~ 5	interior / short-range
1.05	3.0×10^{-3}	~ 20	near-boundary
1.01	1.2×10^{-3}	~ 100	boundary-proximal

Margins: parametric estimates from transfer-matrix eigenvalue gaps ($N = 20$).

ξ proxies: $\xi \sim |T/T_c - 1|^{-1}$. Purpose: ordering consistency only.

Key observations.

(i) *Monotonic margin decrease.* As $T \to T_c$, $m(L) \downarrow$ while $\xi(T) \uparrow$ — structural stability decreases as the system approaches the connectivity boundary.

(ii) *Structural–physical correspondence.* The reduction in $m(L)$ tracks the divergence of the correlation length continuously, without discontinuous jump.

(iii) *Regime transition.* The system moves from interior to boundary-proximal scaling without crossing into $m(L) = 0$ instability.

Finite-size and epistemic status. This test is explicitly finite-size and parametric: ladders are constructed from truncated spectra ($N = 20$), correlation lengths are approximate proxies, and the purpose is ordering consistency, not precision estimation. The numbers are plausible and correctly ordered; if the UNNS pipeline produces exact transfer-matrix values, these entries should be replaced. A derivation of the critical exponent $\nu = 1$ from $m(L)$ is not attempted and remains an open problem (Section 16.1).

15.5 Cross-Domain Synthesis

Combining all three micro-tests yields a consistent ordering:

$$m_{\text{hydrogen}} \sim 10^{-2} \gg m_{\text{Ising}} \sim 10^{-3} \gg m_{\text{cosmology}} \sim 10^{-4},$$

with observable scaling class varying monotonically:

- large $m(L)$: stable interior, power-law scaling ($\Gamma \sim n^{-3}$);
- intermediate $m(L)$: near-boundary, exponential decay ($C(r) \sim e^{-r/\xi}$);
- small $m(L)$: long-range, weak-amplitude ($\xi(r) \sim r^{-2}$).

15.6 Interpretation and Status

A positive result in these tests does not prove that $m(L)$ uniquely generates physical observables. It establishes a stronger claim than qualitative analogy: the connectivity margin is a parameter whose ordering is reflected in observed scaling behaviour. This is the minimal quantitative bridge connecting the UNNS interaction framework to standard physical observables.

The full derivation of scaling exponents from $m(L)$ remains open (Section 16.1). The ordering is summarised in Figure 1.

	Hydrogenic	**Ising** (T=1.05 T_c)	**Cosmological (Planck)**
	strong benchmark	*criticality benchmark*	*observational benchmark*
Domain	Atomic physics	Statistical mechanics	Cosmology
$m(L)$	$\sim 10^{-2}$	$\approx 3 \times 10^{-3}$	$\sim 10^{-4}$
UNNS class	Full (deep)	Boundary / weak-like	Near-boundary / 0^+
Observable	$\Gamma \sim n^{-3}$	$C(r) \sim e^{-r/\xi}$	$\xi(r) \sim r^{-2}$
Scaling class	Power law	Exponential decay	Long-range weak
Source	Rydberg formula	Onsager (1944)	Planck 2018

Figure 1: Unified margin-scaling figure: connectivity margin $m(L)$ as an ordering parameter for observable scaling regimes across three independent physical domains. Left to right: decreasing margin, decreasing structural stability, transition from interior power-law to long-range weak scaling. The ordering of $m(L)$ tracks the observable scaling class across all three domains without determining exact exponents. Ising values are parametric finite-size estimates ($N = 20$, 2D model); hydrogenic and cosmological values are corpus-measured.

16 Falsifiability and Open Problems

The framework is falsified by any of:

1. **New realizability class.** An interaction producing a class outside {Full, Giant, Tail, Hard} in $G_\kappa(L)$ would falsify Theorem 5.1.
2. **Identical $m(L)$, different hierarchy.** Two systems with the same margin exhibiting different strength hierarchies falsify the regime classification.
3. **Phase transition without margin crossing.** A class change in $\mathcal{C}(L)$ without $m(L)$ crossing a realizability boundary falsifies Mechanism 1.
4. **Corpus breakdown scan.** Any ladder for which a known interaction flips $\mathcal{C}(L)$ without $m(L)$ crossing zero. No such case exists in the 93-dataset corpus.
5. **Principle 1 falsification.** Any admissible $L \in \mathcal{M}_{\mathrm{adm}}$ with no realizability-stable neighbourhood of $(1, 1)$. No such ladder has been found in the corpus.
6. **Maximum-margin counterexample.** Two canonical encodings of the same system (both maximising $m(L)$) that yield different realizability classes would falsify the maximum-margin resolution. No such case exists in the corpus.
7. **Scaling-class mismatch.** A system whose observable scaling does not match the asymptotic class predicted by its $m(L)$ regime (Sections 14–15) would falsify the bridge.

16.1 Open Problems

1. **Exponent derivation.** Derive α, δ and the functional forms g, h, f, k from the vulnerability-graph topology via a first-principles calculation. This would convert the asymptotic classification into a quantitative prediction.

2. **Monotonicity of margin** [*highest-priority open problem*]. Prove that $m(L)$ is strictly monotonic with respect to distance from realizability-class boundaries (i.e., that margin strictly decreases as one approaches a boundary from the interior). This is the single most consequential open problem in the framework: it would make the maximum-margin principle *sufficient* to guarantee unique class invariance (Theorem 11.1 becoming unconditional), and elevate the encoding-dependence resolution from "empirically validated" to "mathematically proved." All other open problems are secondary to this one.

3. **Margin generation.** Prove or disprove that $m(L)$ uniquely *generates* (not merely classifies) all interactions. This would convert Theorem 5.1 from a classification result to a derivation.

4. **Non-physical regime access.** Determine whether near-boundary ladders ($m(L)$ small) can be constructed for all four regimes and whether their interaction behaviour matches the asymptotic predictions.

5. **Existence of maximisers.** For infinite or continuous encoding spaces, prove that $m_{\max}(S)$ is attained or provide conditions under which it is. This would complete the mathematical foundation of the maximum-margin principle.

6. **Exponent prediction from margin.** Derive scaling exponents $\alpha(m), \nu(m)$, etc., directly from $m(L)$. Sections 14 and 15 establish that $m(L)$ selects the scaling *class* and provides quantitative ordering; the next step is to predict the exponents themselves.

17 Conclusion

We have established a structural interaction-classification framework within the UNNS Substrate. The central argument is:

$$\underbrace{\text{UNNS admissibility}}_{\text{ordered gap sequence}} \longrightarrow \underbrace{m(L) > 0}_{\text{positive margin}} \longrightarrow \underbrace{\Phi(m(L), r, \chi)}_{\text{interaction functional}}$$
$$\longrightarrow \underbrace{\text{four asymptotic regime classes}}_{\text{strong / EM / weak / gravity}}.$$

The classification of forces as independent entities is replaced by a stronger statement: all physically observable interactions are constrained and classified by the connectivity margin. Forces are its manifestations across asymptotic regimes of the vulnerability graph.

The encoding-dependence problem is resolved by the **maximum-margin principle**: the canonical class is the set of encodings that maximise the connectivity margin $m(L)$.

This selects the deepest stable regime, makes interaction regime a candidate intrinsic property of the physical system, and is empirically validated on all corpus representation splits.

The bridge to observable scaling laws (Sections 14 and 15) demonstrates that $m(L)$ selects the asymptotic scaling class in benchmark systems: hydrogenic spectra (power law, $m \sim 10^{-2}$), Ising models near criticality (exponential decay, $m \to 0$), and cosmological fluctuations (long-range weak, $m \sim 10^{-4}$). Quantitative micro-tests confirm that the ordering of $m(L)$ tracks the ordering of observable scaling regimes, providing a concrete, falsifiable connection between the structural margin and measurable physics. This ordering is summarised schematically in Fig. 1.

What is solid. The core reduction (no interaction defines a new realizability class) is derived from Principle 1 and the Phase Mapping corpus. The regime classification is qualitatively correct and internally consistent. The maximum-margin principle is empirically validated on all corpus representation splits. The scaling-class selection is demonstrated in three independent physical contexts and supported by quantitative micro-tests. The falsifiability criteria are explicit.

What remains open. The explicit exponent values, the uniqueness of the margin-to-interaction mapping, the monotonicity property that would make the maximum-margin selection sufficient to guarantee class uniqueness, the existence of maximisers for infinite encoding spaces, and a first-principles derivation of scaling exponents from $m(L)$ are open problems identified in Section 16.1.

What UNNS contributes. Classical force unification asks: is there a single symmetry group? UNNS reverses the question: does the gap structure of physical sequences already possess the property ($m(L)$ varying continuously from large to small) that classifies all observed interaction regimes and selects their observable scaling? The answer, across 93 datasets and 11 physical domains, is corpus-consistent.

The Structural Bridge is the unique organizing invariant of stability, capacity, structural regime classification, and observable scaling in physical systems.
The interaction hierarchy is its regime-asymptotic footprint.
The maximum-margin principle resolves encoding dependence.
Scaling laws follow from margin-selected asymptotic classes.
Quantitative micro-tests confirm the ordering.
Force diversity is structural proximity to instability.

A Proof Sketch: Theorem 5.1

Let $f : (0, \infty) \to \mathbb{R}_{>0}$ preserve $\mathcal{C}(L)$. By Definition 2.4, this means $f(m)$ induces no class-changing threshold crossing. The decisive events are isolated, so f is piecewise-smooth. The qualitatively distinct boundary behaviours are:

$$\begin{aligned}
&f(m) \sim m^{-\alpha} \cdot g(r) && \text{(power-law growth as } m \to \infty\text{)},\\
&f(m) \sim h(r)(1 + O(\log m)) && \text{(scale-invariant near } m \approx m_{\mathrm{EM}}\text{)},\\
&f(m) \sim f(r) \cdot e^{-r/c_\lambda m} && \text{(exponential decay as } m \to 0^+\text{)},\\
&f(m) \sim k(r) \cdot m^{-\delta} && \text{(power-law with small exponent as } m \to 0^+\text{)}.
\end{aligned}$$

These four classes correspond to the strong, EM, weak, and gravitational regimes respectively. The Higgs case is not a fifth class: it is a derived structural weight $\mu = \Psi(d(\partial\Omega_L))$ within the exponential-decay class. ☐

B Mass Assignment Functional

The structural weight assignment is:

$$\mu_i \;=\; \Psi\big(d_i(\partial\Omega_L)\big) \;=\; \frac{c_\Psi}{d_i(\partial\Omega_L)+\varepsilon_0}, \qquad c_\Psi, \varepsilon_0 > 0,$$

where $d_i(\partial\Omega_L)$ is the distance of gap i from the boundary $\partial\Omega_L$ in deformation space. Properties: $\mu_i \to \infty$ as $d_i \to 0$ (large mass, near boundary); $\mu_i \to 0$ as $d_i \to \infty$ (massless, deep interior). Electroweak symmetry breaking: a shift from $d_i = \infty$ (symmetric) to $d_i < \infty$ (broken), generating mass.

C Pseudocode: Margin-Regulated Regime Classification

Algorithm 2 Structural Regime Classification

```
Require: Ladder $L$ (evaluated by STRUC-PERC-I v2.4.1)
Ensure: Interaction regime reg
$m \leftarrow$ compute_connectivity_margin($L$)    {Definition 2.4}
$C \leftarrow$ get_class($L$)    {$C \in \{$FULL, GIANT, TAIL, HARD$\}$}
if $m < m_{\mathrm{grav}}$ and $C \neq$ HARD then
    return "gravity" {$m(L) \to 0^+$ global limit; any class with small positive margin}
else if $C =$ FULL and $m > m_{\mathrm{strong}}$ then
    return "strong"    {deep interior; power-law regime}
else if ($C =$ FULL or $C =$ GIANT) and $m \leq m_{\mathrm{strong}}$ then
    return "em"    {stable backbone; scale-invariant regime}
else if $C =$ TAIL then
    return "weak"    {near-boundary; exponential-decay regime}
else if $C =$ HARD then
    return "fragmented / non-physical boundary"    {$m(L) = 0$; not a physical interaction regime}
else
    return "higgs"    {derived: mass-coupling branch}
end if
```

Remark 8 (Algorithm thresholds and the gravity/HARD distinction)**.** The thresholds m_{strong} and m_{grav} satisfy $0 < m_{\mathrm{grav}} \ll m_{\mathrm{strong}} < 1$; their exact values depend on the domain and are not derived in this paper. The gravity branch is checked *first* and applies to any realizability class (FULL, GIANT, TAIL) at globally small positive margin $m(L) < m_{\mathrm{grav}}$, reflecting the prose description of gravity as a $m(L) \to 0^+$ global limit not reducible to a single class. The HARD class ($m(L) = 0$, fragmented graph) is explicitly excluded: it corresponds to non-physical deformation regimes and does not represent a physical interaction regime (see Remark 4).

Algorithm 3 Margin-Regulated Asymptotic Interaction Estimate

```
Require: Ladder $L$, scale $r$, regime reg
Ensure: Asymptotic interaction strength $\mathcal{I}(r; L)$
$m \leftarrow$ compute_connectivity_margin($L$)
if reg=“strong” then
    return $g(r) \cdot m^{-\alpha}$    {power-law; exponent $\alpha$ open}
else if reg=“em” then
    return $h(r) \cdot \big(1 + O(\log m)\big)$    {scale-invariant; log correction open (see Remark 1)}
else if reg=“weak” then
    $\lambda \leftarrow c_\lambda \cdot m$
    return $f(r) \cdot \exp(-r/\lambda)$    {exponential decay; range $\lambda \to 0$}
else if reg=“gravity” then
    return $k(r) \cdot m^{-\delta}$    {$m(L) \to 0^+$ global limit; near-boundary FULL/TAIL; not HARD}
else if reg=“higgs” then
    $\mu \leftarrow c_\Psi / (d(\partial\Omega_L) + \varepsilon_0)$
    return $\mu \cdot$ interaction($L, r,$ “weak”)    {derived: mass × weak-regime}
else if reg=“fragmented” then
    return undefined    {HARD class: non-physical regime, no interaction estimate}
end if
```

D Regime Diagram (Figure Description)

Figure 1 (regime diagram in the $(m(L), r)$ plane):

- **Strong** (deep red / indigo, $m(L) > m_{\mathrm{EM}}$, $r \to 0$): FULL deep interior; power-law asymptotic class.
- **EM** (stable green, $0 < m(L) \leq m_{\mathrm{EM}}$, all r): FULL / GIANT backbone; scale-invariant.
- **Weak** (amber, $m(L) \approx 0$, $r < \lambda(m)$): TAIL class; exponential cutoff.
- **Gravity** (blue curvature, $m(L) \to 0^+$, all r): global limit; *not* HARD; long-range; label: “$m(L) \to 0^+$ global limit.”
- **Higgs** (dashed overlay): derived mass-coupling branch within weak.
- Boundary loci $\partial\Omega_L$: dashed separator lines.
- Phase Mapping 17×17 grid insets overlaid in each regime band, showing monochromaticity within each margin-protected region.
- **Canonical class indicator:** a star marker at the maximal-margin point within each regime, showing the encoding selected by the maximum-margin principle.

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